RQuanTech
Boosting the Qubit Revolution

# QUANTUM COMPUTATIONAL FINANCE

## Quantum algorithm for the Monte Carlo pricing of financial derivatives.

**RQuanTech** has developed an innovative and disruptive quantum algorithm for use in financial applications.

In this presentation we show how the relevant probability distributions can be prepared in quantum superposition, the payoff functions can be implemented via **RQuanTech** quantum algorithms, and the price of financial derivatives can be extracted via quantum measurements. We show how the amplitude estimation algorithm can be applied to achieve a quadratic quantum speedup in the number of steps required to obtain an estimate for the price with high confidence.

### INTRODUCTION

A great amount of computational resources is employed by participants in today's financial markets.

Some of these resources are spent on the pricing and risk management of financial assets and their derivatives. Financial assets include the usual stocks, bonds, and commodities, based upon which more complex contracts such as financial derivatives are constructed. Financial derivatives are contracts that have a future payoff dependent upon the future price or the price trajectory of one or more underlying benchmark assets. For these derivatives, due to the stochastic nature of underlying assets, an important issue is the assignment of a fair price based on available information from the markets, what in short can be called the pricing problem. The famous Black-Scholes-Merton (BSM) model can price a variety of financial derivatives via a simple and analytically solvable model that uses a small number of input parameters. A large amount of research has been devoted to extending the BSM model to include complicated payoff functions and complex models for the underlying stochastic asset dynamics.

Monte Carlo methods have a long history in the sciences. Some of the earliest known applications were made in the context of the Los Alamos project, which used early computational devices such as the ENIAC. For the pricing problem in finance, the main challenge is to compute an expectation value of a function of one or more underlying stochastic financial assets. For models beyond BSM, such pricing is often performed via Monte Carlo evaluation.

Quantum computing promises algorithmic speedups for a variety of tasks, such as factoring or optimization.

The Quantum Fourier Algorithm jointly with Shor's algorithm implemented by **RQuanTech** could be extended and generalized to function optimization, amplitude amplification and estimation, integration, quantum walk-based methods for element distinctness, and Markov chain algorithms, for example. In particular, the amplitude estimation algorithm can provide close to quadratic speedups for estimating expectation values, and thus provides a speedup to a problem for which Monte Carlo methods are used classically.

**RQuanTech** presents a new perspective of how to use quantum computing for the pricing problem. We combine well-known quantum techniques, such as amplitude estimation and the quantum algorithm for Monte Carlo with the pricing of financial derivatives.

We first show how to obtain the expectation value of a financial derivative as the output of a quantum algorithm. To this end, we show the ingredients required to set up the financial problem on a quantum computer: the elementary arithmetic operations to compute payoff functions, the preparation of the model probability distributions used in finance, and the ingredients for estimating the expectation value through an imprinted phase on ancilla qubits. It is shown how to obtain the quadratic speedup via the amplitude estimation algorithm.

We provide evidence that a quadratic speedup in pricing can be attained.

**CLASSICAL MONTE CARLO PRICING**

Monte Carlo pricing of financial derivatives proceeds in the following way. Assume that the risk-neutral probability distribution is known or can be obtained from calibrating to market variables. Sample from this risk-neutral probability distribution a market outcome, compute the asset prices given that market outcome, then compute the option payoff given the asset prices.

Averaging the payoff over multiple samples obtains an approximation of the derivative price.

Assume a European option on a single benchmark asset and let the true option price be P and P' be the approximation obtained from *k* samples. Assume that the random variable of the payoff $f(S_T)$ is bounded in variance,

i.e. $V[f(S_T)] < \lambda^2$. Then the probability that the price estimation P' is ε away from the true price is determined by Chebyshev's inequality

$\text{Prob}[|P' - P| \geq \varepsilon] \leq \lambda^2 / k \varepsilon^2$

For a constant success probability, we thus require

$k = O(\lambda^2 / \varepsilon^2)$

samples to estimate to additive error ε. The task of the quantum algorithm will be to improve the ε dependence from $\varepsilon^2$ to ε , hence providing a quadratic speedup for a given error.

Before we present the quantum algorithm for derivative pricing, we show how to encode expectation values into a quantum algorithm and how to obtain the same ε dependency as the classical algorithm. We then show the quadratic speedup by using the fundamental quantum algorithm of amplitude estimation.

**QUANTUM ALGORITHM FOR MONTE CARLO**

Using amplitude estimation for the quantum Monte Carlo pricing of financial derivatives.

(a) The n + 1 qubit phase estimation unitary is written in terms of $F := R(A \otimes I_2)$, and the simple rotation Unitaries, $Z := |I_2^{n+1}> - 2\,|0_{n+}1><0_{n+1}|$ and $V := |I_2^{n+1}> - 2\, I_2^{n} \otimes |1><1|$.

(b) A visualization of the action of $Q := US$, with $S = VUV$ and $U = FZF^\dagger$, on an arbitrary state

$|\psi>$ in the span of χ and *V (χ)*. First, the action of $-S$ on $|\psi>$ is to reflect along *V (χ)*, resulting in the intermediate $-S|\psi>$. Then, $-U$ acts on $-S|\psi>$ by reflecting along

|χ> The resultant state *Q* |χ> has been rotated anticlockwise by an angle 2θ in the hyperplane of |χ> and |χ†>.

(c) The phase estimation circuit. Here, *A* encodes the randomness by preparing a superposition in |x>, while *R* encodes the random variable into the |1> state of an ancilla qubit. The output after both steps is the multiqubit state |χ>. Amplitude estimation then proceeds by invoking phase estimation to encode the rotation angle θ in a register of quantum bits that are measured to obtain the estimate ^ θ.

(d) For pricing a European call option, the superposition prepared by *A* (or equivalently *G*) is a discretization of the normal distribution in x with a fixed cut-off (e.g. c = 4), approximating the Brownian motion of the underlying asset. In this case, *R* encodes the call option payoff.

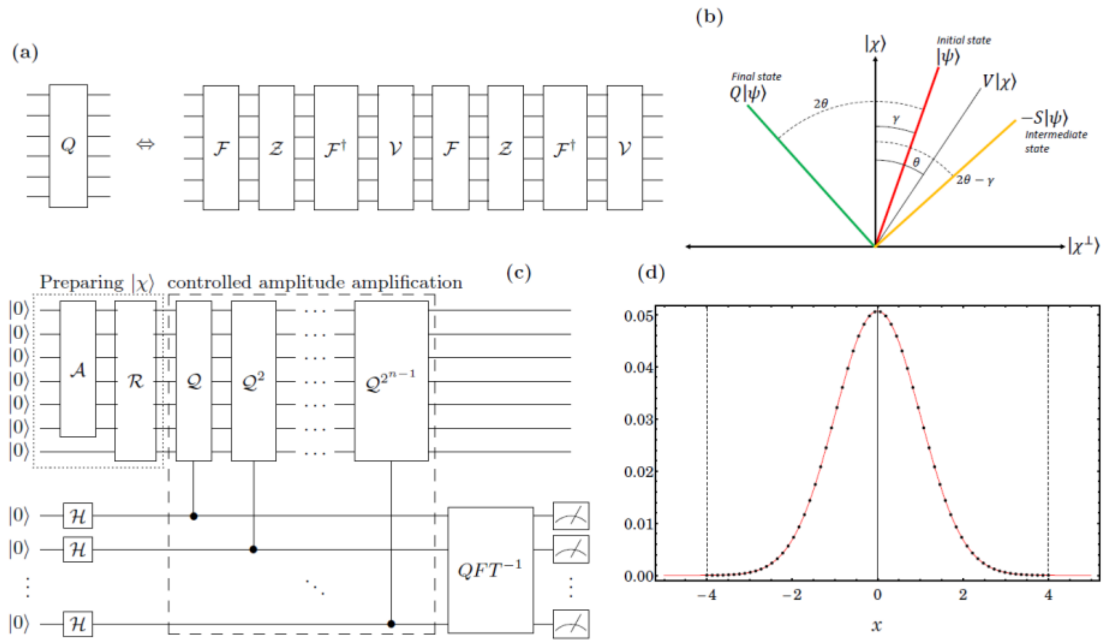


FIG. 1 Algorithm scheme

**CONCLUSION**

We have described a quantum algorithm for the pricing of financial derivatives. We have assumed that the distribution of the underlying random variables, i.e. the martingale measure, is known and the corresponding quantum states can be prepared efficiently.

In addition, we assume efficient computability of the derivative payoff function. Under these assumptions, we exhibit a quadratic speedup in the number of samples required to estimate the price of the derivative up to a given error: if the desired accuracy is $\varepsilon$, then classical methods show a $1 / \varepsilon^2$ dependency in the number of samples, while **RQuanTech** quantum algorithm shows a $1 / \varepsilon$ dependency.

**FURTHER CONSIDERATIONS**

Monte Carlo simulations play a major role in managing the risk that a financial institution is exposed to. Especially after the financial crisis of 2008-9, sophisticated risk management is increasingly important to financial institutions internally and also required by government regulators.

Such risk analysis falls under the umbrella of so-called valuation adjustments (VA), or XVA where X stands for the type of risk under consideration. An example is CVA, where the counterparty credit risk is modelled. Such a valuation adjusts the price of the derivative based on the risk that the counterparty in that financial contract runs out of money.

XVA calculations are a major computational effort for groups (`desks') at financial institutions that handle complex derivatives such as those based on interest rates. For complex financial derivatives, such risk management involves a large amount of Monte Carlo simulations. Different Monte Carlo runs assess the price of a derivative under various scenarios. Determining the risk of the complete portfolio of a desk often requires overnight calculations of the prices according to various risk scenarios.

**RQuanTech** Quantum algorithm promise a significant speedup for such computations. In principle, overnight calculations could be reduced to much shorter time scales (such as minutes), which would allow a more real time analysis of risk. Such close-to real time analysis would allow your firm to react faster to changing market conditions and to profit from trading opportunities.

**RQuanTech** could aid your firm in maximizing your opportunities in this brave new quantum world.